The Christian Mathematician

Devotions for you and your students

Sharon K. Robbert, Ph.D.
Trinity Christian College

A message from Dr. Robbert:

How does one find a meaningful connection between Christianity and mathematics? I believe that there are two approaches to the connection which form an inverse relationship, in a mathematical sense. These are:

- What does it mean to have a Christian perspective of mathematics?
- How does thinking mathematically help to interpret Christian beliefs?

With any problem pair having an inverse relationship, one problem usually is more difficult than the other. The easier problem is labeled "direct" while the more difficult problem is labeled "inverse." My thesis is that to use mathematical thinking to understand the concepts of theological principles is the direct problem; development of a Christian perspective of mathematics is the inverse problem.

Characterization of the issue into these two directions helps me to give context for materials I created as devotionals for mathematics classes at Trinity. For several years I have been noting ways in which mathematics could effectively model theology. I found mathematical concepts in sermons, in scripture readings, and in Christian radio broadcasts. However, it wasn't until I taught an 8:00 a.m. statistics class that I decided to take action in an organized and deliberate manner--devotions for that class would be instances where the mathematics we were studying reflected God. The success in that statistics class motivated me to continue the project with an 8:00 a.m. multivariable calculus class. In 2001, I decided to do weekly devotions for all of my classes with the same ground rules.

This web-resource is a collection of these mathematical reflections of God. I hope that you find something interesting or inspirational in this collection. Feel free to adapt them for your own use. If you have suggestions for improvement or an idea for another devotional that is connected to a topic in an undergraduate mathematics course, please contact me at sharon.robbert@trnty.edu. I welcome additions to the collection.

If you are interested in tackling the more difficult problem, there are two important resources you should consult. One is a recent publication by Eerdmans, Mathematics in a Postmodern Age: A Christian Perspective (ISBN: 0-8028-4910-5). A second is an annotated bibliography of publications which addresses the connection between mathematics and Christianity. This resource was created by members of the Association of Christians in the Mathematical Sciences.

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SINGLE VARIABLE CALCULUS

Mathematical Modeling of Physical Phenomena

Should We Be Surprised?

> *Psalm 8:3-4 NIV*
>
> *When I consider your heavens, the work of your fingers, the moon and the stars, which you have set in place, what is mankind that you are mindful of them, human beings that you care for them?*
>
> *Genesis 1:31 NIV*
>
> *God saw all that he had made, and it was very good. And there was evening, and there was morning—the sixth day.*
>
> *1 Corinthians 13:12 NIV*
>
> *For now we see only a reflection as in a mirror; then we shall see face to face. Now I know in part; then I shall know fully, even as I am fully known.*

Wow! Look at that! How amazing that mathematical equations can be effective at modeling the physical world. But is it truly amazing? If we believe that God created order out of something formless and empty, then what is surprising about finding order in our world?

Maybe we should be surprised (but also grateful) that God included people in His creation who are able to appreciate the beauty of his world. However, we should not be so arrogant that we think our mathematical models can capture the entire essence of His complex creation.

We are told that *now we know in part, a poor reflection; but then we shall know fully!* What will be amazing is knowing fully at the time of Christ's return!

Machine Graphics Flaws and Trick Images

Tools to Prevent Deceptions

> *Matthew 24:24 NIV*
>
> *For false messiahs and false prophets will appear and perform great signs and wonders to deceive, if possible, even the elect.*
>
> *2 Corinthians 11:13-15 NIV*
>
> *For such people are false apostles, deceitful workers, masquerading as apostles of Christ. And no wonder, for Satan himself masquerades as an angel of light. It is not surprising, then, if his servants also masquerade as servants of righteousness. Their end will be what their actions deserve.*
>
> *Ephesians 6:10-17 NKJV*
>
> *Finally, my brethren, be strong in the Lord and in the power of His might. Put on the whole armor of God, that you may be able to stand against the wiles of the devil. For we do not wrestle against flesh and blood, but against principalities, against powers, against the rulers of the darkness of this age, against spiritual hosts of wickedness in the heavenly places. Therefore take up the whole armor of God, that you may be able to withstand in the evil day, and having done all, to stand. Stand therefore, having girded your waist with truth, having put on the breastplate of righteousness, and having shod your feet with the preparation of the gospel of peace; above all, taking the shield of faith with which you will be able to quench all the fiery darts of the wicked one. And take the helmet of salvation, and the sword of the Spirit, which is the word of God;*

In calculus we use technology freely; in particular to produce graphical images with graphing calculators and computer algebra systems. Technology is not perfect, however, and those who use technology must be aware of times when the graphical images we see are not representative of the true nature of the object. We use mathematical experience and developed intuition to judge whether an image is flawed or deceptive.

Satan is the angel of light and his disciples masquerade as "servants of righteousness." [2 Corinthians 11:13--15] But we read in Matthew 24:24 that it is impossible for false Christs to deceive the elect. We must follow the example of Jesus and use scripture as a standard against which to measure truth. We must also put on the full armor of God to protect ourselves from Satan's attacks. In both situations, knowledge helps prevent deception.

Function Operations

God's Surgical Improvements of our Actions

> *Genesis 50:15-21 NKJV*
>
> *When Joseph's brothers saw that their father was dead, they said, "Perhaps Joseph will hate us, and may actually repay us for all the evil which we did to him."*
>
> *So they sent messengers to Joseph, saying, "Before your father died he commanded, saying, 'Thus you shall say to Joseph: "I beg you, please forgive the trespass of your brothers and their sin; for they did evil to you."' Now, please, forgive the trespass of the servants of the God of your father." And Joseph wept when they spoke to him.*
>
> *Then his brothers also went and fell down before his face, and they said, "Behold, we are your servants."*
>
> *Joseph said to them, "Do not be afraid, for am I in the place of God? But as for you, you meant evil against me; but God meant it for good, in order to bring it about as it is this day, to save many people alive. Now therefore, do not be afraid; I will provide for you and your little ones." And he comforted them and spoke kindly to them.*
>
> *Romans 3:9-10 NIV*
>
> *What shall we conclude then? Do we have any advantage? Not at all! For we have already made the charge that Jews and Gentiles alike are all under the power of sin. As it is written: "There is no one righteous, not even one;*
>
> *Romans 3:21-24 NIV*
>
> *But now apart from the law the righteousness of God has been made known, to which the Law and the Prophets testify. This righteousness is given through faith in Jesus Christ to all who believe. There is no difference between Jew and Gentile, for all have sinned and fall short of the glory of God, and all are justified freely by his grace through the redemption that came by Christ Jesus.*

One of the more horrible images in the book of Genesis is that of Joseph being sold by his brothers into slavery. This type of hate turned into evil act is a common occurrence in our world, too. In the Genesis situation, though, we are given the gift of 20-20 hindsight because we know the end of the story. God used the brothers' evil action to prevent starvation of the descendants of Abraham. Joseph says, "You intended to harm me, but God intended it for good. . ." [Genesis 50:20] In the same way, God makes our unrighteous actions righteous through Christ. He surgically improves our actions to his own purpose.

This idea of twisting something from one form into another is what happens when function operations work on elementary functions. You can start with two ordinary benign functions, the reciprocal function 1/x and sin(x), say, and put them together. Depending on how you put them together, you can create something interesting and easily understood, like sin(x)/x, or something with wild behavior, like sin(1/x). Either way, you have twisted one object into something very different.

Development of the Derivative

Does God Change?

> *Psalms 11:3 NIV*
>
> *When the foundations are being destroyed, what can the righteous do?*
>
> *Psalms 107:1 NIV*
>
> *Give thanks to the lord, for he is good; his love endures forever.*
>
> *Psalms 118:1 NIV*
>
> *Give thanks to the lord, for he is good; his love endures forever.*
>
> *Psalms 117:2 NIV*
>
> *For great is his love toward us, and the faithfulness of the lord endures forever. Praise the lord.*
>
> *Lamentations 5:19 NIV*
>
> *You, lord, reign forever; your throne endures from generation to generation.*
>
> *Daniel 4:34 NIV*
>
> *At the end of that time, I, Nebuchadnezzar, raised my eyes toward heaven, and my sanity was restored. Then I praised the Most High; I honored and glorified him who lives forever. His dominion is an eternal dominion; his kingdom endures from generation to generation.*

To understand the concept of derivative, it can be helpful to begin with simple functions. Linear functions, in particular, are very good ones with which to start because their rate of change is always constant (i.e., the slope of the line). Constant functions also have a constant rate of change--the rate of change of any constant function is zero.

So what is God's rate of change function with respect to time? Does he have volatile rate function, one which changes wildly with small increments of time?

NO! We are told that God is steadfast and endures forever. His rate of change is zero--in righteousness, in renown, in love, in faithfulness, in his reign. What a comfort to us to know that God is constant, that He will always be the same from generation to generation, everlasting!

Derivative as a Rate of Change

Indicators for a Person's Heart:

James 2:12-18 NIV

Speak and act as those who are going to be judged by the law that gives freedom, because judgment without mercy will be shown to anyone who has not been merciful. Mercy triumphs over judgment.

What good is it, my brothers and sisters, if someone claims to have faith but has no deeds? Can such faith save them? Suppose a brother or a sister is without clothes and daily food. If one of you says to them, "Go in peace; keep warm and well fed," but does nothing about their physical needs, what good is it? In the same way, faith by itself, if it is not accompanied by action, is dead.

But someone will say, "You have faith; I have deeds." Show me your faith without deeds, and I will show you my faith by my deeds.

Which comes first--the function or the derivative? If you are given a function in a graphical, symbolical, or numerical representation, you can find all the information you need about the derivative of that function--at least in the same representation.

On the other hand, if you are given a derivative of a function, you can once again find out information about the original function. Is the original function increasing or decreasing? Is it concave up or down? But you will not know everything--you cannot know the exact location of the function vertically. That information is lost when the derivative is calculated.

Now compare a person's heart to their actions. If you know a person's heart, you should be able to predict actions completely, assuming consistency that is. But, suppose you only know a person's actions? Will you know that person's heart completely?

Of course, only God can see the true nature of the person's heart. Human observers can only observe the behavior and then predict the nature of the heart. What conclusions will observers make when they see actions that do not model Christ? Their conclusions will inevitably be that the heart is not faithful to God. James says that "faith by itself, if it is not accompanied by action, is dead." So we must model Christ faithfully in our actions. This is the natural outcome of faith!

Limit Definition of a Derivative

Secant Lines and Sanctification:

Psalms 119:33-40 NIV

Teach me, lord, the way of your decrees,
that I may follow it to the end.
Give me understanding, so that I may keep your law
and obey it with all my heart.
Direct me in the path of your commands,
for there I find delight.
Turn my heart toward your statutes
and not toward selfish gain.
Turn my eyes away from worthless things;
preserve my life according to your word.
Fulfill your promise to your servant,
so that you may be feared.
Take away the disgrace I dread,
for your laws are good.
How I long for your precepts!
In your righteousness preserve my life.

In differential calculus we study how a slope of a linear function can be generalized to the slope of a function whose graph is curved, creating the derivative of the original function. The definition of derivative uses a sequence of lines (secant lines) drawn through two points on a function that are approaching each other and a single point on the function curve. The derivative value or tangent line slope is defined to be the limiting slope value of this sequence of secant lines. (See the figures below.)

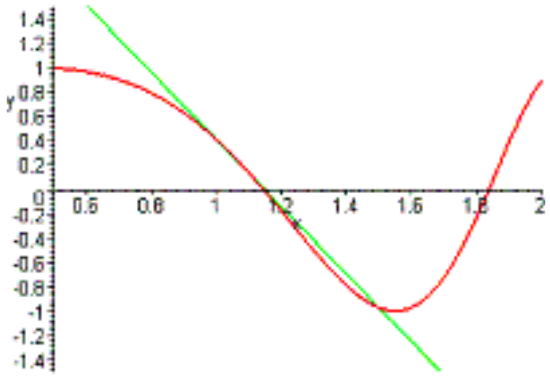


Figure 1 : Secant line between 1 and 1.8

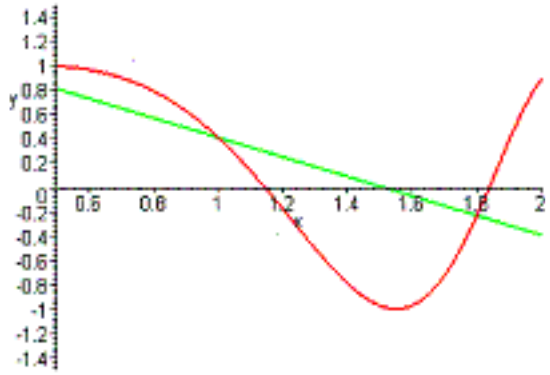


Figure 2 : Secant line between 1 and 1.

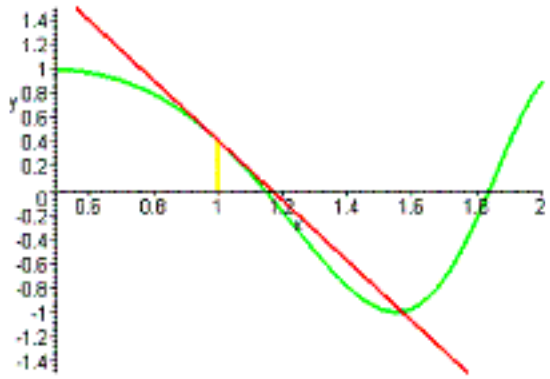


Figure 3: Tangent line to f at x=1

Once a person has been called to be a Christian, we are redeemed by Christ but not released from following the law of God. We are justified once but continue with the process of sanctification for the remainder of our lives. This sanctification process is like the limit process of the secant lines approaching the tangent line.

There is one distinction between the concepts of sanctification and secant line limits, however. In the mathematical contexts, we accept results that are "sufficiently close," results that are in an epsilon-neighborhood of the desired quantity. While in our quest for perfection, the "better" we get the further we realize we are from satisfying all aspects of the law.

Infinite Limits

Infinite Qualities of God:

Psalms 139:1-18 NIV

You have searched me, lord,
and you know me.
You know when I sit and when I rise;
you perceive my thoughts from afar.
You discern my going out and my lying down;
you are familiar with all my ways.
Before a word is on my tongue
you, lord, know it completely.
You hem me in behind and before,
and you lay your hand upon me.
Such knowledge is too wonderful for me,
too lofty for me to attain.
Where can I go from your Spirit?
Where can I flee from your presence?
If I go up to the heavens, you are there;
if I make my bed in the depths, you are there.
If I rise on the wings of the dawn,
if I settle on the far side of the sea,
even there your hand will guide me,
your right hand will hold me fast.
If I say, "Surely the darkness will hide me
and the light become night around me,"
even the darkness will not be dark to you;
the night will shine like the day,
for darkness is as light to you.

For you created my inmost being;

you knit me together in my mother's womb.

I praise you because I am fearfully and wonderfully made;

your works are wonderful,

I know that full well.

My frame was not hidden from you

when I was made in the secret place,

when I was woven together in the depths of the earth.

Your eyes saw my unformed body;

all the days ordained for me were written in your book

before one of them came to be.

How precious to me are your thoughts, God!

How vast is the sum of them!

Were I to count them,

they would outnumber the grains of sand—

when I awake, I am still with you.

Infinite limits are a wonderful extension of the mathematical world studied in calculus. In a precalculus context, behavior of a function at a value where the function is not defined is excluded from consideration. (No division by zero nor even roots of negatives allowed!)

But in a calculus context, evaluation of function behavior near these same input values numbers yields interesting characteristics of the function. Do function values approach infinitely large positive results from both slightly larger input values and slightly smaller input values? If so, we say that the limit of the function at this excluded input value is infinite. Disagreement between the two one-directional limits still gives interesting information, but not quite as general as in the case where the two agree.

Limits help finite creatures such as ourselves understand the amazing qualities of our Creator God a little better. Where we have finite knowledge, God has complete knowledge--exceeding the limit of all accumulated human knowledge for all time! Where we are limited to a single presence, God is not--he is everywhere present simultaneously! Where we are weak, God is all powerful; he can control

time (see 2 Kings 20) and weather (see Matt 8:23-27) and physical laws (see Matt 14:22--36), and the hearts of people (see Exodus 7:3-4). Where we have a life-span of at most 100 years, God is not bounded by time; God transcends time (see 2 Peter 3:8).

God exceeds the limits of our understanding in so many ways.

Symbolic Differentiation

Transformation under Christ

Ephesians 2:1-7 NKJV

And you He made alive, who were dead in trespasses and sins, in which you once walked according to the course of this world, according to the prince of the power of the air, the spirit who now works in the sons of disobedience, among whom also we all once conducted ourselves in the lusts of our flesh, fulfilling the desires of the flesh and of the mind, and were by nature children of wrath, just as the others. But God, who is rich in mercy, because of His great love with which He loved us, even when we were dead in trespasses, made us alive together with Christ (by grace you have been saved), and raised us up together, and made us sit together in the heavenly places in Christ Jesus, that in the ages to come He might show the exceeding riches of His grace in His kindness toward us in Christ Jesus.

Ephesians 4:22-24 NIV

You were taught, with regard to your former way of life, to put off your old self, which is being corrupted by its deceitful desires; to be made new in the attitude of your minds; and to put on the new self, created to be like God in true righteousness and holiness.

Differentiation is an operator on functions that takes one functions and transforms it into another form. The new form is related to the old form--the derivative tells interesting information about how the original function behaves graphically--but it is a completely new function.

When someone accepts Jesus as Lord of their life and gives themselves wholly to God as one of His creatures, a similar transformation occurs. The person is transformed through Jesus and the Holy Spirit into a new self. This new self is redeemed, purified of unrighteousness, and claimed by God for eternal life. The new self is truly new, yet it still retains characteristics of the original self.

God does not want his people to be identical to one another. God's creatures are distinct individuals with unique thoughts, unique gifts, unique appearances, and unique contributions to the body of Christ! Let us rejoice in our diversity in the community of the redeemed!

Chain Rule

Chain Reactions

> *2 Corinthians 5:16-21 NKJV*
>
> *Therefore, from now on, we regard no one according to the flesh. Even though we have known Christ according to the flesh, yet now we know Him thus no longer. Therefore, if anyone is in Christ, he is a new creation; old things have passed away; behold, all things have become new. Now all things are of God, who has reconciled us to Himself through Jesus Christ, and has given us the ministry of reconciliation, that is, that God was in Christ reconciling the world to Himself, not imputing their trespasses to them, and has committed to us the word of reconciliation. Now then, we are ambassadors for Christ, as though God were pleading through us: we implore you on Christ's behalf, be reconciled to God. For He made Him who knew no sin to be sin for us, that we might become the righteousness of God in Him.*

Once students have seen the chain rule for differentiation of composed functions, it is natural to extend the chain rule to nested functions, where there is more than two functions that are composed. Fun problems to investigate are ones that are repeated applications of the same function. Try differentiating tan(tan(tan(tan(tan x)))) or ln(ln(ln(ln(ln(ln x))))), for example. Working your way from the outside to the inside yields a derivative which is product chain of related functions.

In a similar way, when we interact with other people there is a chain reaction to our behavior. Most people believe that abused children are more likely to become abusers themselves someday, for example. Less dramatic behavior also can have a reaction that extends beyond the initial engagement. A popular Warner Brothers film of 2000 Pay it Forward (based on a novel with the same name by Catherine Ryan Hyde) depicts how a chain of reactions to an initial act of kindness can change an entire community. Christians need to be specially mindful of this chain reaction, since we are ambassadors for Christ. Our verbal and nonverbal witness can yield unexpected results, especially under the influence of the Holy Spirit.

Inverse Functions and Derivatives

Inversions in the Bible

> *2 Kings 20:1-11 NIV*
>
> *In those days Hezekiah became ill and was at the point of death. The prophet Isaiah son of Amoz went to him and said, "This is what the lord says: Put your house in order, because you are going to die; you will not recover."*
>
> *Hezekiah turned his face to the wall and prayed to the lord, "Remember, lord, how I have walked before you faithfully and with wholehearted devotion and have done what is good in your eyes." And Hezekiah wept bitterly.*
>
> *Before Isaiah had left the middle court, the word of the lord came to him: "Go back and tell Hezekiah, the ruler of my people, 'This is what the lord, the God of your father David, says: I have heard your prayer and seen your tears; I will heal you. On the third day from now you will go up to the temple of the lord. I will add fifteen years to your life. And I will deliver you and this city from the hand of the king of Assyria. I will defend this city for my sake and for the sake of my servant David.'"*
>
> *Then Isaiah said, "Prepare a poultice of figs." They did so and applied it to the boil, and he recovered.*
>
> *Hezekiah had asked Isaiah, "What will be the sign that the lord will heal me and that I will go up to the temple of the lord on the third day from now?"*
>
> *Isaiah answered, "This is the Lord's sign to you that the lord will do what he has promised: Shall the shadow go forward ten steps, or shall it go back ten steps?"*
>
> *"It is a simple matter for the shadow to go forward ten steps," said Hezekiah. "Rather, have it go back ten steps."*
>
> *Then the prophet Isaiah called on the lord, and the lord made the shadow go back the ten steps it had gone down on the stairway of Ahaz.*

Investigation of real-valued functions on the real numbers includes some common questions. Some of these questions are about the behavior of the function:

- Where does the function increase? decrease?
- Where does the function have an increasing rate of change (i.e., where is the function concave up?)
- Is this function monotone?

- If so, what is the inverse of this function?

- If not, can you find a smaller domain where the function is monotone with range on the modified domain the same as the original function?

- Under these conditions, what is the inverse?

Existence of an inverse function is a very useful property of a function--it is much easier to answer questions about where a function takes on a particular output value IF we can use an inverse function.

Applying inverse functions in nature is something environmentalists know is very difficult to do. Think about how difficult it is to restore a harvested rain forest or to undo the ill effects of an oil spill! However, our God is able to restore nature and he will to do it completely at the second coming of Christ. There are a few examples in the Bible of God's power over nature. God can reverse or stop the spin of the earth (see 2 Kings 20 or Joshua 10). God can make iron float on water (see 2 Kings 6) or stop the flow of a river (see Joshua 3).

Three BIG Theorems: MVT, EVT, IVT

Faith vs. Proof

> *Matthew 8:5-13 NIV*
>
> *When Jesus had entered Capernaum, a centurion came to him, asking for help.*
>
> *"Lord," he said, "my servant lies at home paralyzed, suffering terribly."*
>
> *Jesus said to him, "Shall I come and heal him?"*
>
> *The centurion replied, "Lord, I do not deserve to have you come under my roof. But just say the word, and my servant will be healed. For I myself am a man under authority, with soldiers under me. I tell this one, 'Go,' and he goes; and that one, 'Come,' and he comes. I say to my servant, 'Do this,' and he does it."*
>
> *When Jesus heard this, he was amazed and said to those following him, "Truly I tell you, I have not found anyone in Israel with such great faith. I say to you that many will come from the east and the west, and will take their places at the feast with Abraham, Isaac and Jacob in the kingdom of heaven. But the subjects of the kingdom will be thrown outside, into the darkness, where there will be weeping and gnashing of teeth."*
>
> *Then Jesus said to the centurion, "Go! Let it be done just as you believed it would." And his servant was healed at that moment.*

Do you believe that the big theorems (EVT, IVT, MVT, FTC, etc.) of Calculus are true? Or do you need to see a proof? Some are willing to believe everything a perceived authority says is true, and we might disparage them as gullible people. Each person is different in what they are willing to believe on faith--this depends upon experience, how the new information fits with current knowledge, and their trust in the authority espousing the information.

In mathematics, there is tension between what must be proven and what can be accepted on faith. In Calculus, some of the proofs are too complex for the context of learning the new information while others can be very instructive in understanding a new topic.

In the Christian tradition, much of what we believe must be taken on faith. Jesus was impressed by those who were willing to accept his status as son of God without proof. The centurion identified with the authority of Jesus because of his own experience in the military. We can speculate that he was witness to some of the miracles of Jesus, giving evidence to support his faith. But his willingness to accept

the word of Jesus that his servant would be healed was beyond Jesus' expectations. We hope that Jesus will be impressed by our willingness to believe and that we will be blessed by our faith.

Differential Equations: Growth Functions

Blessed Exponentially

> *Matthew 5:43-47 NIV*
>
> *"You have heard that it was said, 'Love your neighbor and hate your enemy.' But I tell you, love your enemies and pray for those who persecute you, that you may be children of your Father in heaven. He causes his sun to rise on the evil and the good, and sends rain on the righteous and the unrighteous. If you love those who love you, what reward will you get? Are not even the tax collectors doing that? And if you greet only your own people, what are you doing more than others? Do not even pagans do that?"*
>
> *Genesis 12:2-3 NIV*
>
> *"I will make you into a great nation, and I will bless you; I will make your name great, and you will be a blessing. I will bless those who bless you, and whoever curses you I will curse; and all peoples on earth will be blessed through you."*

Elementary functions play an important role in calculus. The rate at which those elementary functions grow for increasing input values is one characteristic we study. [Note: Applications of this growth analysis appear in algorithm complexity analysis in computer science. Exponential growth is "bad" in this instance.] The fastest growing elementary function class is the exponential function; a function which takes various powers of a fixed numerical base. The principle of exponential growth is exploited in savings plans (save early and often!) and modeled in growth of bacteria.

Christ tells us in Matthew 5:43-47 that we are to love our neighbors AND our enemies. We also read in Genesis 12:2-3 that God blessed Abraham so that "all peoples on earth will be blessed through you." Together the concepts of exponential growth and "blessed to be a blessing" tell us to "pay it forward," so that God's love for humankind and goodness can grow exponentially.

Integration as a Measurement Tool

Measurement in the Bible

> *1 Kings 7:23 NIV*
>
> *He made the Sea of cast metal, circular in shape, measuring ten cubits from rim to rim and five cubits high. It took a line of thirty cubits to measure around it.*

> *Genesis 6:15 NIV*
>
> *This is how you are to build it: The ark is to be three hundred cubits long, fifty cubits wide and thirty cubits high.*

> *1 Kings 6:2-3 NIV*
>
> *The temple that King Solomon built for the lord was sixty cubits long, twenty wide and thirty high. The portico at the front of the main hall of the temple extended the width of the temple, that is twenty cubits, and projected ten cubits from the front of the temple.*

Are measurements in the Bible are to be taken literally? If so, then p is exactly three--the Great Sea in Solomon's temple was circular in shape, ten cubits in diameter and 30 cubits in circumference. Most mathematicians take exception with this value for p! Here we must assume that either the measurements or the circular shape are not exact.

Other biblical measurements are also interesting. Noah's ark is 450 feet long by 75 feet wide and 45 feet high, assuming an 18 inch cubit. Compared to an NFL football field, the ark would be 1.25 the length, a little less than half as wide, and 1.5 times the height of the goal post. So the ark wouldn't fit completely on the marked field, but would fit in the central arena of Soldier Field in Chicago. Solomon's temple was much smaller, only 90 feet long and 30 feet wide with a height of 45 feet. Approximately ten copies of this temple would fit in the inside of Noah's ark. Intriguing, isn't it?

In general, I believe that numbers in the Bible are to be taken figuratively, not literally. Knowing that measurement was important enough to the Israelites to be recorded is what I find most interesting. I take comfort in the fact that whether or

not these numbers are accurate does not change the story of God's redemption for people and his creation.

Numeral Integration and Error Bounds

God's Zero Tolerance for Error

> *Philippians 3:1--9 NKJV*
>
> *Finally, my brethren, rejoice in the Lord. For me to write the same things to you is not tedious, but for you it is safe. Beware of dogs, beware of evil workers, beware of the mutilation! For we are the circumcision, who worship God in the Spirit, rejoice in Christ Jesus, and have no confidence in the flesh, though I also might have confidence in the flesh.*
>
> *If anyone else thinks he may have confidence in the flesh, I more so: circumcised the eighth day, of the stock of Israel, of the tribe of Benjamin, a Hebrew of the Hebrews; concerning the law, a Pharisee; concerning zeal, persecuting the church; concerning the righteousness which is in the law, blameless. But what things were gain to me, these I have counted loss for Christ. Yet indeed I also count all things loss for the excellence of the knowledge of Christ Jesus my Lord, for whom I have suffered the loss of all things, and count them as rubbish, that I may gain Christ and be found in Him, not having my own righteousness, which is from the law, but that which is through faith in Christ, the righteousness which is from God by faith;*

Analytically finding the area between a curve and the horizontal axis is a primary topic in integral calculus. We learn that some curves are resistant to exact method of computation, so geometric estimation techniques are required. In every estimation problem, it is insufficient to find an estimate without also knowing theoretically how close the estimate is to the quantity we wish to estimate---this is finding an upper bound on the error. We deal with relations that look something like this:

|Desired Quantity - Estimate| $\leq$ Error bound.

In most applied situations we can allow for a small error; if we're off by 0.00001, that might be okay.

There is an equivalent error analysis when we compare our attempts to meet God's law and the perfection demanded by God's holiness. Here, God requires zero tolerance for error in order to be accepted into His kingdom. So the relation looks like this:

$|\text{Standard of God's Law - Our imperfect actions}| \not\in 0.$

We are unable to meet this zero error bound, so on our own we cannot be accepted into the kingdom. However, Christ exchanged places with us---He put his perfect self in our place in comparison to God's Law and took our punishment of death. This makes us able to satisfy the zero tolerance for error.

MULTIVARIABLE CALCULUS

Planar Approximations and Intentions

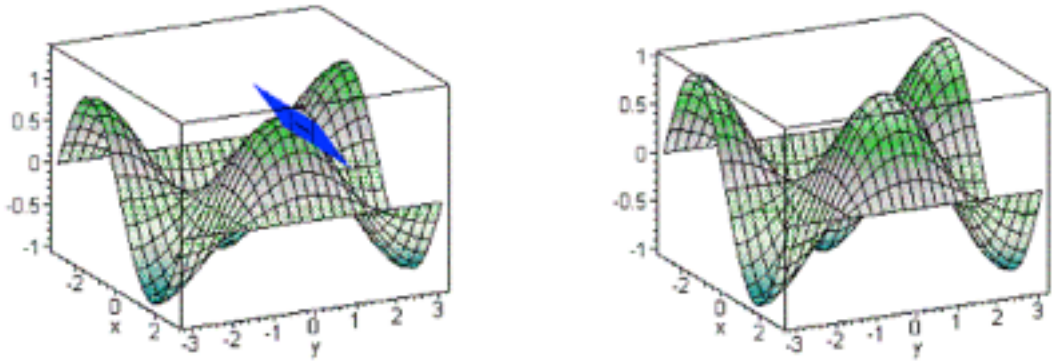
Approximations and Intentions

> *I Corinthians 13:12 NIV*
>
> *For now we see only a reflection as in a mirror; then we shall see face to face. Now I know in part; then I shall know fully, even as I am fully known.*

Sometimes, analysis of a given relationship in multivariable settings is complex--too complex to be worth the effort of exact calculation. Instead, it is sufficient to use an approximation for the relationship that matches some but not all of the essential characteristics of the original relationship.

This is modeled below by the planar approximation for the lumpy surface. To calculate values of the lumpy surface, we instead find a simple function (here a plane) that in a small window around the point the values will agree within some tolerance for error.

A Planar Approximation to a Lumpy Surface

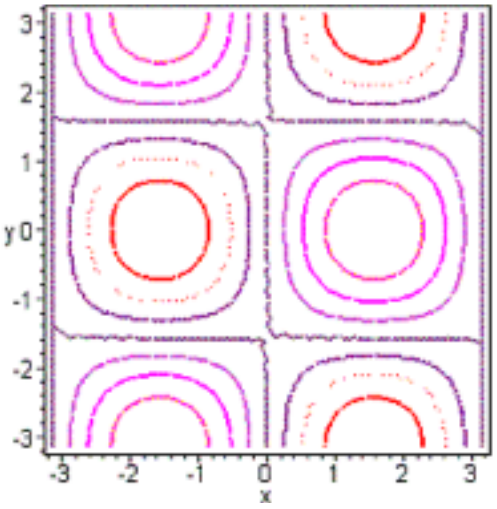


Level Sets

Stratification in Christ's kingdom?

> *Colossians 3:10--11 NIV*
>
> *...and have put on the new self, which is being renewed in knowledge in the image of its Creator. Here there is no Gentile or Jew, circumcised or uncircumcised, barbarian, Scythian, slave or free, but Christ is all, and is in all.*
>
> *Romans 12: 6--8 NIV*
>
> *We have different gifts, according to the grace given to each of us. If your gift is prophesying, then prophesy in accordance with your faith; if it is serving, then serve; if it is teaching, then teach; if it is to encourage, then give encouragement; if it is giving, then give generously; if it is to lead, do it diligently; if it is to show mercy, do it cheerfully.*

In the three-dimensional images you see above, the location of the hills and valleys are easy to see. Not quite as easy to see in the diagram are the saddle points that lay diagonally between two valleys and two hills. To better understand the characteristics of a function, students use level sets, a collection of two-dimensional graphs which gives detail about slices of the function using regularly spaced heights. At left is a portion of the level sets for the lumpy surface above. The hills and the valleys in the level set plot are at the centers of the concentric "circles" while saddle points occur at the intersection of the boundary lines.



This idea of grouping the function input values by output values is a little like the stratification we see daily in human culture. There are those who are perceived to be at the highest level (e.g., Michael Jordan, Queen Elizabeth II) and those who we see at the bottom of the deepest valleys (e.g., death-row inmates, Al Qaeda terrorists). However, we learn in Colossians 3:10--11 that in Christ there is no such ranking. "Here there is no Greek or Jew, ." and all Christians must "put on the new self, which is being renewed in knowledge in the image of its Creator." However, Christians are not clones of each other. Each is given a different configuration of spiritual gifts. In this type of stratification, we rejoice in the distinctive service we can provide for God's kingdom.

DISCRETE STRUCTURES

Overview: Conjecture and Proof

Proof of God's Will

> *Romans 12:1-2 NIV*
>
> *Therefore, I urge you, brothers and sisters, in view of God's mercy, to offer your bodies as a living sacrifice, holy and pleasing to God—this is your true and proper worship. Do not conform to the pattern of this world, but be transformed by the renewing of your mind. Then you will be able to test and approve what God's will is—his good, pleasing and perfect will.*

One thing that developing mathematicians must learn to do is to design new mathematical systems. At the sophomore level, we begin that process within a system where students have a lot of experience---number relationships. Students look at simple patterns and try to generalize the relationships they see. The generalization created is called a conjecture.

Conjectures are excellent first steps in the design of new mathematical systems; however, to be useful, the person must try to write a convincing argument demonstrating why the statement is true or find a counter-example demonstrating why the statement is false.

Developing Christians are taught to seek the will of God in making life-decisions. To be able to determine the direction God wishes us to go, we must form conjectures and reason to conviction of truth. Paul says in Romans 12:2, "be transformed by renewing of your mind. Then you will be able to test [conjecture] and approve [reason to conviction of truth] what God's will is ."

Logical Operators

Epistle Implications

> *1 John 1:5-9 NIV*
>
> *This is the message we have heard from him and declare to you: God is light; in him there is no darkness at all. If we claim to have fellowship with him and yet walk in the darkness, we lie and do not live out the truth. But if we walk in the light, as he is in the light, we have fellowship with one another, and the blood of Jesus, his Son, purifies us from all sin. If we claim to be without sin, we deceive ourselves and the truth is not in us. If we confess our sins, he is faithful and just and will forgive us our sins and purify us from all unrighteousness.*

There are lots of logical implications in the new testament. Read any letter written by Paul, for example, and look for the word "therefore." An important pair of implications is found in 1 John 1:

> vs. 8: If we say we are without sin, [then] we deceive ourselves and the truth is not in us.
>
> vs. 9: If we confess our sins, [then] he is faithful and just and will forgive us our sins and purify us from all unrighteousness.

Note in particular that the verse 9 implication uses the negation of the hypothesis of verse 8 as its premise. It is also interesting that the conclusion of verse 9 shifts emphasis from man to God. These two verses cover all cases in an analysis of how man responds to his relationship to sin: either you deny your sin or you confess your sin.

Logical Quantifiers

Unique Existence of God

Exodus 3:13-14 NIV

Moses said to God, "Suppose I go to the Israelites and say to them, 'The God of your fathers has sent me to you,' and they ask me, 'What is his name?' Then what shall I tell them?" God said to Moses, "i am who i am. This is what you are to say to the Israelites: 'i am has sent me to you.'"

Deuteronomy 6:4-5 NIV

Hear, O Israel: The lord our God, the lord is one. Love the lord your God with all your heart and with all your soul and with all your strength.

Romans 1:18-20 NIV

The wrath of God is being revealed from heaven against all the godlessness and wickedness of people, who suppress the truth by their wickedness, since what may be known about God is plain to them, because God has made it plain to them. For since the creation of the world God's invisible qualities—his eternal power and divine nature—have been clearly seen, being understood from what has been made, so that people are without excuse.

In the process of learning acceptable mathematical procedures for writing an argument which is convincing to other readers, we study predicate logic. Determining what portion of the entire collection will satisfy a relationship is one component of the argument. Mathematicians indicate the special cases of all, at least one, and exactly one with quantifier notation. If an open statement P(x) is true for all valid replacements x, we write "x,P(x). If the open statement P(x) is true for at least one replacement, we write $x,P(x). And, if an open statement is true for one and only one replacement, we write $!x,P(x). Unique existence of a valid replacement is one of the most special cases to consider. A proof of this type of statement always requires two parts: first, you must show that at least one solution exists (i.e., existence of solution); then you must show that not more than one solution exists (i.e., uniqueness of solution).

Abraham and his descendants were chosen to be the first people on earth to be led to comprehend both aspects of the unique existence of God. One instance of the existence portion of God is found in the story of Moses meeting God in the burning bush. Here, God reveals his name to Moses as evidence that He truly exists. God says, "I am who I am." [Note: Exodus 3:13--14. I especially enjoy the transcendence of God to time given within the Hebrew for this phrase--the phrase can be interpreted with past, present, and future verb tenses!]

Later, at Mount Sinai, Moses is given laws to train the infant-nation of Israel in the ways of God. In Deuteronomy 6:4, Moses recounts the uniqueness condition told him by God: "Hear, O Israel: The Lord our God, the Lord is one." Even though both aspects of the "$! God" were provided to Israel from the time of the exodus, we know from Old Testament stories that the lesson was a difficult one for these chosen people to learn. I think even today we struggle with acknowledging God's unique existence, though few Christians will deny the truth of the statement.

Proof writing

A Perfect Proof?

> *James 2:8-11 NIV*
>
> *If you really keep the royal law found in Scripture, "Love your neighbor as yourself," you are doing right. But if you show favoritism, you sin and are convicted by the law as lawbreakers. For whoever keeps the whole law and yet stumbles at just one point is guilty of breaking all of it. For he who said, "You shall not commit adultery," also said, "You shall not murder." If you do not commit adultery but do commit murder, you have become a lawbreaker.*

Suppose you write a proof that is perfect--except at one point where there is a logical flaw. Does the proof hold? Of course the answer is "no." The proof must have absolutely no logical flaws to be accepted as valid. The same is true of people who try to keep the law of God perfectly. If there is even one small part that is not held perfectly, the entire law is broken. Since we cannot keep the law perfectly, our judgment is death apart from Christ.

Recursion and Induction

Recursive Blessings and Curses

> *Exodus 20: 4-6 NIV*
>
> *"You shall not make for yourself an image in the form of anything in heaven above or on the earth beneath or in the waters below. You shall not bow down to them or worship them; for I, the lord your God, am a jealous God, punishing the children for the sin of the parents to the third and fourth generation of those who hate me, but showing love to a thousand generations of those who love me and keep my commandments."*

Recursion and Induction are important topics in mathematics and computer science. In each context, one takes a initial object (a base case or cases) and creates new objects from ones already known with a recursion. For example, to define n! using a recursive definition, one defines 0! to be 1 and says that $(n + 1)! = (n + 1) \times n!$. Or, to prove that a statement is true for all natural numbers using math induction, one first proves that the statement is true for the case $n = 1$, then one proves that IF the case with $n = k$ forces the case with $n = k + 1$ to also be true regardless of the k-value selected, then the statement must be valid for all natural numbers.

It is interesting to note that God also uses the concept of recursion. In the third commandment, do not worship idols, God makes it very clear that he will punish or reward people based on their response to this command. God will recursively punish children for three to four generations for the sin of one person. But His recursive blessings for those who love Him and keep his commandments will go to thousands!

Disjoint Sets

Adopted Children of God

> *Ephesians 1:3-9 NIV*
>
> *Praise be to the God and Father of our Lord Jesus Christ, who has blessed us in the heavenly realms with every spiritual blessing in Christ. For he chose us in him before the creation of the world to be holy and blameless in his sight. In love he predestined us for adoption to sonship through Jesus Christ, in accordance with his pleasure and will— to the praise of his glorious grace, which he has freely given us in the One he loves. In him we have redemption through his blood, the forgiveness of sins, in accordance with the riches of God's grace that he lavished on us. With all wisdom and understanding, he made known to us the mystery of his will according to his good pleasure, which he purposed in Christ.*

> *Two sets are disjoint if they have no elements in common. In this case, we write A ÇB = Æ. Disjoint sets of objects are easy to create; consider, for example, the sets A = {1, 3, 5} and B = {2, 4}. These sets have no numbers in common.*

Okay, so what about people? Are there disjoint sets of people? Many would like to claim that we are separated by race or creed or nationality. But through Christ we know that in God's eyes these are not true distinctions (see Gal 3:28). God does separate people into a pair of disjoint sets (which is actually a partition): people who are redeemed and people who are not redeemed. Those who are redeemed are adopted heirs of salvation through faith in Jesus. However, people are not given the insight to determine who is in each of these sets. It is the commission of redeemed people to spread the good news of salvation to all people and let God sort out who will be in each set.

Definition of Function

The Jesus Function

> *Isaiah 1:18 NIV*
>
> *"Come now, let us settle the matter," says the Lord. "Though your sins are like scarlet, they shall be as white as snow; though they are red as crimson, they shall be like wool."*

A functions is a rule which assigns to each object in a domain set exactly one object in a codomain set. So, suppose that the domain set is the collection of behaviors that people do and the codomain consists of two values: perfect or imperfect.

On our own, every behavior that we do is mapped to the "imperfect" output value. But, the Jesus function takes our behavior and filters it through his sacrifice so that God takes the output of the behavior of Christians as "perfect" in terms of our final judgment. Praise God!

Inverse Functions

$Christ = Adam^{-1}$

1 Corinthians 15:20-23 NIV

But Christ has indeed been raised from the dead, the firstfruits of those who have fallen asleep. For since death came through a man, the resurrection of the dead comes also through a man. For as in Adam all die, so in Christ all will be made alive. 23 But each in turn: Christ, the firstfruits; then, when he comes, those who belong to him.

An inverse mapping of any function reverses the direction of the assignment of the function. In the case that the original function is one-to-one, the inverse mapping will also be a function with the domain and codomain interchanged.

Through the sin of Adam, all people are condemned for eternity. We cannot escape our inherited imperfection. Let's define the "Adam life function" to act on people. This function has output death for all inputs. However, Christ inverts the total condemnation of people through his death and resurrection. This acts as a sort of inverse to the "Adam life function;" the "Christ death function" acts on all people and brings life to those who believe. This is not truly an inverse function--it is more of a negation--the negation of Adam (man) is Christ and the negation of life is death. It is ironic that to give eternal life to people whose lives are undeserving, Christ had to die.

Equivalence Classes

A Common Confession

Romans 10:9-10 NIV

If you declare with your mouth, "Jesus is Lord," and believe in your heart that God raised him from the dead, you will be saved. For it is with your heart that you believe and are justified, and it is with your mouth that you profess your faith and are saved.

Philippians 2:9-11 NIV

Therefore God exalted him to the highest place and gave him the name that is above every name, that at the name of Jesus every knee should bow, in heaven and on earth and under the earth, and every tongue acknowledge that Jesus Christ is Lord, to the glory of God the Father.

A partition of sets is a separation of every element from a universal set into a collection of pairwise disjoint sets. If a valid partition is created, then one can create from that partition an equivalence relation on the universal set. Two objects will be equivalence if they fall in the same subset of the partition. God has partitioned people into two sets: the redeemed and the unredeemed. But, regardless of classification, ALL will someday confess that Jesus is Lord.

Combinations

Countable or Uncountable?

Numbers 1:46 NIV

The total number was 603,550.

1 Chronicles 21:1-14 NKJV

Now Satan stood up against Israel, and moved David to number Israel. So David said to Joab and to the leaders of the people, "Go, number Israel from Beersheba to Dan, and bring the number of them to me that I may know it."

And Joab answered, "May the Lord make His people a hundred times more than they are. But, my lord the king, are they not all my lord's servants? Why then does my lord require this thing? Why should he be a cause of guilt in Israel?"

Nevertheless the king's word prevailed against Joab. Therefore Joab departed and went throughout all Israel and came to Jerusalem. Then Joab gave the sum of the number of the people to David. All Israel had one million one hundred thousand men who drew the sword, and Judah had four hundred and seventy thousand men who drew the sword. But he did not count Levi and Benjamin among them, for the king's word was abominable to Joab.

And God was displeased with this thing; therefore He struck Israel. So David said to God, "I have sinned greatly, because I have done this thing; but now, I pray, take away the iniquity of Your servant, for I have done very foolishly."

Then the Lord spoke to Gad, David's seer, saying, "Go and tell David, saying, 'Thus says the Lord: "I offer you three things; choose one of them for yourself, that I may do it to you." '

" So Gad came to David and said to him, "Thus says the Lord: 'Choose for yourself, either three years of famine, or three months to be defeated by your foes with the sword of your enemies overtaking you, or else for three days the sword of the Lord—the plague in the land, with the angel of the Lord destroying throughout all the territory of Israel.' Now consider what answer I should take back to Him who sent me."

And David said to Gad, "I am in great distress. Please let me fall into the hand of the Lord, for His mercies are very great; but do not let me fall into the hand of man." So the Lord sent a plague upon Israel, and seventy thousand men of Israel fell.

Luke 2:1-3 NIV

In those days Caesar Augustus issued a decree that a census should be taken of the entire Roman world. (This was the first census that took place while Quirinius was governor of Syria.) And everyone went to their own town to register.

Revelation 7:4 NIV

Then I heard the number of those who were sealed: 144,000 from all the tribes of Israel.

Revelation 7:9 NIV

After this I looked, and there before me was a great multitude that no one could count, from every nation, tribe, people and language, standing before the throne and before the Lamb. They were wearing white robes and were holding palm branches in their hands.

There are numerous events where counting occurs in the Bible. The resulting numbers recorded are not necessarily literal, though. Many of the results are figurative. Some of the counting events were performed at God's command while other counts were due to the pride of men.

For example, we read that there are roughly 600 thousand Israelite men at the time of the exodus from Egypt. Later the number of "fighting men" in Israel was counted to be 1.1 million by David the King in a moment of doubt, and then 70 thousand of these men were killed in a plague of judgment. After the exile, 10% of the remnant were chosen to live in Jerusalem. These men numbered 3,044 "brave men." So, the remnant of Israel number in all approximately 30 thousand men. Rome took a census at the time of Jesus birth--though the results are not recorded in scripture. In the vision of John recorded in Revelations, 144 thousand people from the tribes of Israel were given a seal on their foreheads but an uncountable number in white robes from "every nation, tribe, people and language" before the throne of the Lamb!

STATISTICS

Probability Odds

Probability Defied

Judges 7:2-8 NIV

The Lord said to Gideon, "You have too many men. I cannot deliver Midian into their hands, or Israel would boast against me, 'My own strength has saved me.' Now announce to the army, 'Anyone who trembles with fear may turn back and leave Mount Gilead.'" So twenty-two thousand men left, while ten thousand remained.

But the Lord said to Gideon, "There are still too many men. Take them down to the water, and I will thin them out for you there. If I say, 'This one shall go with you,' he shall go; but if I say, 'This one shall not go with you,' he shall not go."

So Gideon took the men down to the water. There the Lord told him, "Separate those who lap the water with their tongues as a dog laps from those who kneel down to drink." Three hundred of them drank from cupped hands, lapping like dogs. All the rest got down on their knees to drink.

The Lord said to Gideon, "With the three hundred men that lapped I will save you and give the Midianites into your hands. Let all the others go home."

So Gideon sent the rest of the Israelites home but kept the three hundred, who took over the provisions and trumpets of the others.

Matthew 19:16-26 NIV

Just then a man came up to Jesus and asked, "Teacher, what good thing must I do to get eternal life?"

"Why do you ask me about what is good?" Jesus replied. "There is only One who is good. If you want to enter life, keep the commandments."

"Which ones?" he inquired.

Jesus replied, "'You shall not murder, you shall not commit adultery, you shall not steal, you shall not give false testimony, honor your father and mother,' and 'love your neighbor as yourself.'"

"All these I have kept," the young man said. "What do I still lack?"

Jesus answered, "If you want to be perfect, go, sell your possessions and give to the poor, and you will have treasure in heaven. Then come, follow me."

When the young man heard this, he went away sad, because he had great wealth.

> *Then Jesus said to his disciples, "Truly I tell you, it is hard for someone who is rich to enter the kingdom of heaven. Again I tell you, it is easier for a camel to go through the eye of a needle than for someone who is rich to enter the kingdom of God."*
>
> *When the disciples heard this, they were greatly astonished and asked, "Who then can be saved?"*
>
> *Jesus looked at them and said, "With man this is impossible, but with God all things are possible."*

The probability of success is determined mathematically by looking at the empirical ratio (number of successes)/(number of possibilities), where the number of successes and possibilities are taken from a theoretical sample space. For example, if you want to calculate the probability that you will roll a seven with a pair of distinct dice, you note that there are 6 ways to roll a seven (1-6, 2-5, 3-4, 4-3, 5-2, and 6-1) from among all the 36 possible combinations. So the probability of rolling a seven is 6/36 or 1/6. This ratio is always a number between 0 and 1, inclusive. The larger the number in this interval, the more likely the event is to occur. Less likely events have a smaller number in the interval.

God is able to beat the odds, however. He proved to Gideon that this is the case by systematically eliminating a large portion of his fighting men, selecting only 300 out of 32,000 to fight the Midianites. Jesus also used probabilities to teach his disciples. He says in Matthew 19:24 that "it is easier for a camel to go through the eye of a needle than for a rich man to enter the kingdom of God." [Note: some Biblical scholars take the eye of the needle to be a small but busy gate into the city of Jerusalem.] So, the probability of salvation for a rich man is given as very small. He comforts his disciples in verse 26 of the same chapter by letting them know that God is more powerful than probabilities---"with God all things are possible."

Measures of Central Tendency

Don't Be Average

> *Revelations 3:14-22 NIV*
>
> *"To the angel of the church in Laodicea write:*
> *These are the words of the Amen, the faithful and true witness, the ruler of God's creation. I know your deeds, that you are neither cold nor hot. I wish you were either one or the other! So, because you are lukewarm—neither hot nor cold—I am about to spit you out of my mouth. You say, 'I am rich; I have acquired wealth and do not need a thing.' But you do not realize that you are wretched, pitiful, poor, blind and naked.I counsel you to buy from me gold refined in the fire, so you can become rich; and white clothes to wear, so you can cover your shameful nakedness; and salve to put on your eyes, so you can see.*
>
> *Those whom I love I rebuke and discipline. So be earnest and repent. Here I am! I stand at the door and knock. If anyone hears my voice and opens the door, I will come in and eat with that person, and they with me.*
>
> *To the one who is victorious, I will give the right to sit with me on my throne, just as I was victorious and sat down with my Father on his throne. Whoever has ears, let them hear what the Spirit says to the churches."*

When we study numerical data in statistics, we organize the data, draw visual summaries, rank the data, and calculate measures of central tendency, among other things. With measures of central tendency, we calculate the mean (arithmetic average) and the median (middle value) to try to get a meaningful way to describe the data. Most teachers use mean and median and other descriptive statistics on test results to decide if students are performing as we expect in a class. Most students are happy if they are near or above the middle of the pack on a test score.

However, God want more than a middling performance in terms of our Christian behavior. He wants our behavior to be far away from that of the average "good" person. In Revelations 3:14--22, John is told to write to the Laodicean church that their lukewarm performance is causing them to be in danger of eternal rejection. Likewise, we must not be content with our current state as a Christian. We must always work to be even more "extreme" for God.

Central Limit Theorem

Repeated Samples of Character

Deuteronomy 4:32-40 NKJV

For ask now concerning the days that are past, which were before you, since the day that God created man on the earth, and ask from one end of heaven to the other, whether any great thing like this has happened, or anything like it has been heard. Did any people ever hear the voice of God speaking out of the midst of the fire, as you have heard, and live? Or did God ever try to go and take for Himself a nation from the midst of another nation, by trials, by signs, by wonders, by war, by a mighty hand and an outstretched arm, and by great terrors, according to all that the Lord your God did for you in Egypt before your eyes? To you it was shown, that you might know that the Lord Himself is God; there is none other besides Him. Out of heaven He let you hear His voice, that He might instruct you; on earth He showed you His great fire, and you heard His words out of the midst of the fire. And because He loved your fathers, therefore He chose their descendants after them; and He brought you out of Egypt with His Presence, with His mighty power, driving out from before you nations greater and mightier than you, to bring you in, to give you their land as an inheritance, as it is this day. Therefore know this day, and consider it in your heart, that the Lord Himself is God in heaven above and on the earth beneath; there is no other. You shall therefore keep His statutes and His commandments which I command you today, that it may go well with you and with your children after you, and that you may prolong your days in the land which the Lord your God is giving you for all time.

Judges 3:7 NIV

The Israelites did evil in the eyes of the Lord; they forgot the Lord their God and served the Baals and the Asherahs.

Judges 3:12 NIV

Again the Israelites did evil in the eyes of the Lord, and because they did this evil The Lord gave Eglon king of Moab power over Israel.

Judges 4:1 NIV

Again the Israelites did evil in the eyes of the Lord, now that Ehud was dead.

Judges 10:6 NIV

The Israelites did evil in the eyes of the Lord, and for seven years he gave them into the hands of the Midianites.

Judges 13:1 NIV

Again the Israelites did evil in the eyes of the Lord, so the Lord delivered them into the hands of the Philistines for forty years.

Repeated sampling of the character of Israel demonstrated evil tendency. But a repeated sampling of the character of Jesus demonstrated his divinity. Among the accounts of his miracles, there are twenty-three healing events, eight power over nature events, and four resurrections. Surely Yahweh is truly God!

LINEAR ALGEBRA

Spanning Set

The Span of God's Love

> *Ephesians 3:14-21 NIV*
>
> *For this reason I kneel before the Father, from whom every family in heaven and on earth derives its name. I pray that out of his glorious riches he may strengthen you with power through his Spirit in your inner being, so that Christ may dwell in your hearts through faith. And I pray that you, being rooted and established in love, may have power, together with all the Lord's holy people, to grasp how wide and long and high and deep is the love of Christ, and to know this love that surpasses knowledge—that you may be filled to the measure of all the fullness of God. Now to him who is able to do immeasurably more than all we ask or imagine, according to his power that is at work within us, to him be glory in the church and in Christ Jesus throughout all generations, for ever and ever! Amen.*

The span of a collection of vectors is defined to be the set of all possible linear combinations of the vectors with scalars selected from some set (a field--usually the real numbers). For example, if you select vectors i = <1, 0> and j = <0, 1>, the collection of linear combinations ai + bj (for a, b real numbers) is every vector in the two-dimensional coordinate plane. Creating something so large using something so small is surprising but at the same time expected because of previous experience with vectors, say in Calculus.

Now let's think about something whose words are very simple, say the love of God, and meditate on what the span of God's love might be. We read in Paul's letters to the Ephesians that he prays that they might "have the power. . . to grasp how wide and long and high and deep is the love of Christ." So the span of God's love is really really big! Is this surprising? YES! Just think about our rebellion and sin--so it is a surprise that God should love us! Is this expected? Again, yes! We have models in our families and friends that demonstrate how broad a father's love for his children can be.

"Now to him who is able to do immeasurably more that all we ask or imagine. . . be glory . . throughout all generations, for ever and ever!"

Independent and Dependent Vectors

Independent or Dependent?

1 Corinthians 12:7-26 NKJV

But the manifestation of the Spirit is given to each one for the profit of all: for to one is given the word of wisdom through the Spirit, to another the word of knowledge through the same Spirit, to another faith by the same Spirit, to another gifts of healings by the same Spirit, to another the working of miracles, to another prophecy, to another discerning of spirits, to another different kinds of tongues, to another the interpretation of tongues. But one and the same Spirit works all these things, distributing to each one individually as He wills. For as the body is one and has many members, but all the members of that one body, being many, are one body, so also is Christ. For by one Spirit we were all baptized into one body— whether Jews or Greeks, whether slaves or free— and have all been made to drink into one Spirit. For in fact the body is not one member but many. If the foot should say, "Because I am not a hand, I am not of the body," is it therefore not of the body? And if the ear should say, "Because I am not an eye, I am not of the body," is it therefore not of the body? If the whole body were an eye, where would be the hearing? If the whole were hearing, where would be the smelling? But now God has set the members, each one of them, in the body just as He pleased. And if they were all one member, where would the body be? But now indeed there are many members, yet one body. And the eye cannot say to the hand, "I have no need of you"; nor again the head to the feet, "I have no need of you." No, much rather, those members of the body which seem to be weaker are necessary. And those members of the body which we think to be less honorable, on these we bestow greater honor; and our unpresentable parts have greater modesty, but our presentable parts have no need. But God composed the body, having given greater honor to that part which lacks it, that there should be no schism in the body, but that the members should have the same care for one another. And if one member suffers, all the members suffer with it; or if one member is honored, all the members rejoice with it.

One way to demonstrate that a collection of vectors is dependent is to show that one vector in the set can be written as a nontrivial linear combination of other vectors in the set. In this case, the one vector can be eliminated from the set without loss of any vectors from the span of the set. If eliminating any single vector from the set changes the span of the set, then that vector is not a linear combination of the remaining. Students quickly learn that a set of vectors cannot be dependent and independent at the same time--if a set of vectors is independent, there is no reason to continue searching for a way to eliminate a vector from the set. All are essential in that case.

Christians form an independent collection in the church, too. Each person is given a unique set of spiritual gifts that is essential in completing the church community. However, when you read 1 Corinthians 12, it is quickly apparent that this essential to the whole type of independence does not mean "independent" in a colloquial sense of the word--each is still dependent upon all the others in the church community to fulfill the mission God has ordained for that group. To create the same "span," each individual is needed and all need the others. Again, all are essential!

Matrix Transformation

Transformed by the Spirit

> *2 Corinthians 3:18 NIV*
>
> *And we all, who with unveiled faces contemplate the Lord's glory, are being transformed into his image with ever-increasing glory, which comes from the Lord, who is the Spirit.*

> *Isaiah 1:18 NIV*
>
> *"Come now, let us settle the matter," says the lord. "Though your sins are like scarlet, they shall be as white as snow; though they are red as crimson, they shall be like wool."*

Matrix transformations are really fun. You can take an image, define the edges by particular vectors, and then transform or deform the image into new shapes. It is even possible to change the dimension of the space in which a vector lives if you use a matrix which has non-square dimensions.

God promises to transform us, too. We begin as distorted creatures--"our sins are like scarlet!" But through salvation in Jesus, our sins will become "white as snow!" The transformation of our status as guilty to innocent occurs immediately so we "reflect the Lord's glory" today! (Justification) At the same time, the transformation to be in the likeness of Christ is not immediate. We are told in 2 Corinthians 3 that we are in the process of being transformed "with ever-increasing glory" through the Spirit. (Sanctification!)

Matrix Inversions

Our Inverted Actions

Romans 7:14-25 NIV *Romans 8:1-4 NIV*

We know that the law is spiritual; but I am unspiritual, sold as a slave to sin. 1I do not understand what I do. For what I want to do I do not do, but what I hate I do. And if I do what I do not want to do, I agree that the law is good. As it is, it is no longer I myself who do it, but it is sin living in me. For I know that good itself does not dwell in me, that is, in my sinful nature. For I have the desire to do what is good, but I cannot carry it out. For I do not do the good I want to do, but the evil I do not want to do—this I keep on doing. Now if I do what I do not want to do, it is no longer I who do it, but it is sin living in me that does it.

So I find this law at work: Although I want to do good, evil is right there with me. For in my inner being I delight in God's law; but I see another law at work in me, waging war against the law of my mind and making me a prisoner of the law of sin at work within me. What a wretched man I am! Who will rescue me from this body that is subject to death?

Thanks be to God, who delivers me through Jesus Christ our Lord! So then, I myself in my mind am a slave to God's law, but in my sinful nature a slave to the law of sin.

Therefore, there is now no condemnation for those who are in Christ Jesus, because through Christ Jesus the law of the Spirit who gives life has set you free from the law of sin and death. For what the law was powerless to do because it was weakened by the flesh, God did by sending his own Son in the likeness of sinful flesh to be a sin offering. And so he condemned sin in the flesh, in order that the righteous requirement of the law might be fully met in us, who do not live according to the flesh but according to the Spirit.

There is a really easy method to solve a system of equations in matrix form, theoretically speaking. If you multiply both sides of the matrix equation by the inverse of the coefficient matrix, the solution is immediately found. Of course, there is a catch--finding the inverse of the coefficient matrix can be computationally difficult or even impossible. The theoretical value is significant, though. Knowing whether or not a solution exists helps a mathematician decide which method to attempt!

Theoretically, it is also easy to talk about how one can become pure--just "invert" your actions and stop breaking the Law of God. The catch this time is that it is

impossible for humans to stop sinning. Paul admits that he is a "sold as a slave to sin." He continues, "For what I want to do I do not do, but what I hate I do." Fortunately for us, Jesus Christ set us free from the law of sin and death. Christ solves the equation for us!

Determinants

The Mark of a Determinant

Ezekiel 9:1-7 NKJV

Then He called out in my hearing with a loud voice, saying, "Let those who have charge over the city draw near, each with a deadly weapon in his hand." And suddenly six men came from the direction of the upper gate, which faces north, each with his battle-ax in his hand. One man among them was clothed with linen and had a writer's inkhorn at his side. They went in and stood beside the bronze altar.

Now the glory of the God of Israel had gone up from the cherub, where it had been, to the threshold of the temple. And He called to the man clothed with linen, who had the writer's inkhorn at his side; and the Lord said to him, "Go through the midst of the city, through the midst of Jerusalem, and put a mark on the foreheads of the men who sigh and cry over all the abominations that are done within it."

To the others He said in my hearing, "Go after him through the city and kill; do not let your eye spare, nor have any pity. Utterly slay old and young men, maidens and little children and women; but do not come near anyone on whom is the mark; and begin at My sanctuary."

So they began with the elders who were before the temple. Then He said to them, "Defile the temple, and fill the courts with the slain. Go out!" And they went out and killed in the city.

Galatians 6:17 NIV

From now on, let no one cause me trouble, for I bear on my body the marks of Jesus.

Revelations 7:3 NIV

"Do not harm the land or the sea or the trees until we put a seal on the foreheads of the servants of our God."

Revelations 13:16 NIV

It also forced all people, great and small, rich and poor, free and slave, to receive a mark on their right hands or on their foreheads,

Systems of equations play an extremely important role in applied mathematics. A system of equations is a set of equations that are solved in tandem; solutions to the system must satisfy every equation individually. In these systems, complex relationships can be modeled. Relationships such as those between sectors of the United States economy, components of computer-aided design, and even the flight controls of the space shuttle can be modeled and examined in simulation---all

without putting drivers of cars and astronauts at risk. One method that mathematicians can use to determine whether a systems has a solution or not is by calculation of a matrix determinant number. If the determinant value is not zero, then a unique solution to the system exists. Though the determinant has theoretical value, its practical value is limited. It often takes more computational effort to find the value of the determinant than it does to apply common system solution algorithms.

This type of indicator exists in scripture as well--there are spiritual marks that indicate the bearer's allegiance to God or to Satan. Most people are familiar with the "mark of the beast" John describes in his vision in the book of Revelations. Here the mark of the beast was the number 666. However, there are several instances where God marks the faithful with a mark or a seal. In a vision, Ezekiel hears God instruct his assistant, "the man clothed in linen," to put a mark on the foreheads of those who have stayed allied with God. These persons were to be spared execution when God's vengeance was delivered. Other instances of marks or seals of God are found in Galatians 6:17 and in Revelations 7:3. Except for the marks of Christ Paul describes in the Galatians passage, these marks are not practical; they only occur in a spiritual setting. This makes the connection to the determinant more striking.

Generalized Vector Space

Generalization and Fulfillment

Matthew 5:17-20 NIV

> *"Do not think that I have come to abolish the Law or the Prophets; I have not come to abolish them but to fulfill them. For truly I tell you, until heaven and earth disappear, not the smallest letter, not the least stroke of a pen, will by any means disappear from the Law until everything is accomplished. Therefore anyone who sets aside one of the least of these commands and teaches others accordingly will be called least in the kingdom of heaven, but whoever practices and teaches these commands will be called great in the kingdom of heaven. For I tell you that unless your righteousness surpasses that of the Pharisees and the teachers of the law, you will certainly not enter the kingdom of heaven."*

Many concepts in mathematics are studied in a spiral. We study a concept in a conceptual setting, then generalize to a more complex setting by increasing dimension, and then generalize again to an even more complex setting by selecting characteristics that appear useful and then removing all other aspects of the conceptual setting. One of the first times the second type of generalization occurs in the undergraduate mathematics curriculum is in the study of general vector spaces.

[Note: Instances of this type of generalization also occur in non-Euclidean geometry, in group theory, etc.] Students find the lack of a conceptual setting troubling; in fact, instructors of linear algebra often refer to students "hitting the wall" when they first encounter general vector spaces.

Christ also had trouble with his students "hitting the wall" during his ministry. One important aspect of Jesus incarnation was to teach people the meaning behind the law of Moses. His death and resurrection are described as the fulfillment of the Old Testament law. This generalization of the law is beautifully described by Jesus during the sermon on the mount in Matthew 5-7. A common phrase Jesus used during this sermon is "You have heard that it was said.." Each of these phrases is followed by a generalization of an Old Testament law to include the intent behind the law: murder is generalized to include hatred, adultery is generalized to incorporate lust, and love for neighbors is generalized to love for all. The students of Jesus who "hit the wall" were the ones who thought they understood the law the best--the leaders of the Jewish faith.

Bases of a Subspace

THE Basis of the Law

> *Mark 12:28-34*
>
> *One of the teachers of the law came and heard them debating. Noticing that Jesus had given them a good answer, he asked him, "Of all the commandments, which is the most important?"*
>
> *"The most important one," answered Jesus, "is this: 'Hear, O Israel: The Lord our God, the Lord is one. Love the Lord your God with all your heart and with all your soul and with all your mind and with all your strength.' The second is this: 'Love your neighbor as yourself.' There is no commandment greater than these."*
>
> *"Well said, teacher," the man replied. "You are right in saying that God is one and there is no other but him. To love him with all your heart, with all your understanding and with all your strength, and to love your neighbor as yourself is more important than all burnt offerings and sacrifices."*
>
> *When Jesus saw that he had answered wisely, he said to him, "You are not far from the kingdom of God." And from then on no one dared ask him any more questions.*
>
> *Deuteronomy 6:4*
>
> *Hear, O Israel: The lord our God, the lord is one. Love the lord your God with all your heart and with all your soul and with all your strength.*
>
> *Leviticus 19:18*
>
> *"'Do not seek revenge or bear a grudge against anyone among your people, but love your neighbor as yourself. I am the lord.'"*

A basis for a vector space or a subspace of a vector space is a linearly independent collection of vectors whose span is the entire space. To find one basis for a subspace of Euclidean space which is the columnspace of a matrix, one looks for the largest collection of columns of the original matrix that are linearly independent. These columns are a set of essential vectors--from them one is able to recover the entire columnspace of the original matrix.

Basis appear in the Bible, too. To help the Israelite nation become worthy of the title, chosen people, God gave them a set of laws to follow. Although we often focus on the Ten Commandments, the Old Testament Israelites had many laws to follow. (See Leviticus, for example.) God knew that the people of Israel needed lots of specifics, but he also knew that remembering every detail would be tough.

So he gave them a summary, "Love the Lord your God with all your heart and with all your soul and with all of your mind and with all your strength."

By the time Jesus came to earth, the Israelite people had forgotten that the intent of the law was based in the love of God and were focused on the details. So, Jesus reminded them that the BASIS for the law was this summary and that this summary captures all of the essential elements of the law of God.

Vector Space Dimension

Dimension of God

Psalms 139:1-10 NIV

You have searched me, Lord,
and you know me.
You know when I sit and when I rise;
you perceive my thoughts from afar.
You discern my going out and my lying down;
you are familiar with all my ways.
Before a word is on my tongue
you, Lord, know it completely.
You hem me in behind and before,
and you lay your hand upon me.
Such knowledge is too wonderful for me,
too lofty for me to attain.
Where can I go from your Spirit?
Where can I flee from your presence?
If I go up to the heavens, you are there;
if I make my bed in the depths, you are there.
If I rise on the wings of the dawn,
if I settle on the far side of the sea,
even there your hand will guide me,
your right hand will hold me fast.

John 20:19 NIV

On the evening of that first day of the week, when the disciples were together, with the doors locked for fear of the Jewish leaders, Jesus came and stood among them and said, "Peace be with you!"

The dimension of a vector space is a constant which indicates the number of vectors in any basis for the vector space. This number is independent of which

basis is selected. The vector space of three-dimensional space has dimension three (surprise, surprise). However, not all vector spaces have a finite dimension. If you think of the vector space consisting of polynomials with real number coefficients, bases have a countably infinite number of elements. It is easy to construct and infinite basis here; the standard basis is the infinite collection of polynomials 1, x, x2, x3, . . . xn, . . .

God also is infinite dimensional, very likely even of uncountable dimension! There is no way to escape God's presence! If we think of God as an infinite-dimensional being, we can extend the concepts from the book Flatland* to understand a little of the nature of Christ's post-resurrection body. In that book, creatures in a higher-dimensional space are able to enter and exit lower-dimensional spaces easily. The image to think about is a sphere crossing a particular fixed plane. If the sphere is outside the plane, it cannot be seen by objects living only in the plane. But if the sphere crosses the plane, it will appear as a point or an expanding/contracting circle. In the same way, if we think of Christ living in a higher dimension, his sudden appearance in a locked room makes a little more sense. Broadening the concept of dimension is certainly helpful in this sense.

*You can read Flatland online at http://www.alcyone.com/max/lit/flatland/.

Eigenspaces

In the Eigenspace of Christ

2 Corinthians 3:7-18 NKJV

But if the ministry of death, written and engraved on stones, was glorious, so that the children of Israel could not look steadily at the face of Moses because of the glory of his countenance, which glory was passing away, how will the ministry of the Spirit not be more glorious? For if the ministry of condemnation had glory, the ministry of righteousness exceeds much more in glory. For even what was made glorious had no glory in this respect, because of the glory that excels. For if what is passing away was glorious, what remains is much more glorious. Therefore, since we have such hope, we use great boldness of speech— unlike Moses, who put a veil over his face so that the children of Israel could not look steadily at the end of what was passing away. But their minds were blinded. For until this day the same veil remains unlifted in the reading of the Old Testament, because the veil is taken away in Christ. But even to this day, when Moses is read, a veil lies on their heart. Nevertheless when one turns to the Lord, the veil is taken away. Now the Lord is the Spirit; and where the Spirit of the Lord is, there is liberty. But we all, with unveiled face, beholding as in a mirror the glory of the Lord, are being transformed into the same image from glory to glory, just as by the Spirit of the Lord.

The eigenspace of an eigenvalue l is the collection of all vectors u that are mapped to lu under the action of a fixed matrix. It is important to note that u choices exclude the zero vector because the zero vector always is mapped to itself under this type of transformation. On our own, we are like the zero vector because no matter what we try, we cannot move away from out sinful status. However, through the grace of Christ, we are transformed from being a zero vector to the eigenspace of the redeemed (the likeness of Christ).

Eigenvalues

What is your Eigen?

> *Romans 1:8-10 NIV*
>
> *First, I thank my God through Jesus Christ for all of you, because your faith is being reported all over the world. God, whom I serve in my spirit in preaching the gospel of his Son, is my witness how constantly I remember you in my prayers at all times; and I pray that now at last by God's will the way may be opened for me to come to you.*

"Eigen" is a cool word, much more fun the the English version "character." Finding an eigenvalue sounds so much more exotic that finding a characteristic value. But, it is still true that the value you find represents in an important way the action of linear transformation given by a matrix. The character/eigen of the matrix is important in applications like least-squares solutions and differential equations. In a way, knowing the eigen of a matrix is a way of completely characterizing what the matrix is able to do.

So how do you determine the "eigen" of a person? The eigen of a person is evident in the actions that the person does--others should be able to classify your eigen through daily interactions with you in routine settings. A demonstration of a Christ-like eigen is essential to spreading the good news of salvation. Paul writes to the Christians in Rome that he will "give thanks . . your character (eigen) is reported around the world." Let's do the same today!

Least-Squares Solution

Our Inconsistent System

> *Romans 3:21-24 NIV*
>
> *But now apart from the law the righteousness of God has been made known, to which the Law and the Prophets testify. This righteousness is given through faith in Jesus Christ to all who believe. There is no difference between Jew and Gentile, for all have sinned and fall short of the glory of God, and all are justified freely by his grace through the redemption that came by Christ Jesus.*

Least-squares solutions techniques are used when a system has no actual solution; that is, the situation where a system is inconsistent. In this case, we find an object that is the closest to being a solution among all possibilities. This object minimizes the value of the distance between the transformation of the object and the impossible result.

In our lives we must solve an impossible problem--we must perfectly meet the entire law of God if we are to have eternal life. No matter how hard we try, we are unable to do this. Fortunately, God loves us so much that he sent his Son Jesus to solve the problem for us. Christ took upon himself our sin and gave his life so that we might live. Jesus is our least-squares solution to the impossible problem. Note, though, that the distance between us and the law of God is infinite; through salvation in Christ, God accepts us as if the error is zero!